Kolmogorov operators: local and nonlocal models – February 21, 2024

10:00 - 10:20	Opening (Room 155/10, DIISM)
10:20 – 11:00	Sergio Polidoro (UniMoRe) "Taylor formulas for Nonlocal Kinetic Equations"
11:05 – 11:45	Alessia E. Kogoj (UniUrb) "Liouville theorems for Kolmogorov-Fokker-Planck equations"
11:50 – 12:30	Giampiero Palatucci (UniPr) "Boundedness estimates and Harnack inequalities for kinetic integral equations"
12:30 - 14:00	Lunch break
14:05 – 14:45	Stefano Biagi (PoliMi) "KFP operators with coefficients measurable in time and Dini continuous in space"
14:50 – 15:30	Mirco Piccinini (UniPi) "On the lack of compactness in the critical Sobolev embedding in the Heisenberg group"
15:35 – 16:15	Giulio Tralli (UniFe) "Gaining symmetries in the Heisenberg group: the case of bounded domains"

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Stefano Biagi (Politecnico di Milano)

"KFP operators with coefficients measurable in time and Dini continuous in space"

In this talk we consider (possibly) degenerate Kolmogorov-Fokker-Planck operators of the following non-divergence form

$$\begin{aligned} \mathcal{L}u &= \sum_{i,j=1}^{m_0} a_{ij}(x,t) \partial_{x_i x_j}^2 u + \langle Bx, \nabla_x u \rangle - \partial_t u = \sum_{i,j=1}^{m_0} a_{ij}(x,t) \partial_{x_i x_j}^2 u + Y u \\ & (\text{with } (x,t) \in \mathbb{R}^{N+1} \text{ and } 1 \leq m_0 \leq N), \end{aligned}$$

where the coefficients $a_{ij}(x,t)$ are Dini-continuous in space but only bounded in time, and the matrix B takes a suitable lower-triangular block form. Under these assumptions, we prove global partial a-priori estimates, controlling the "partial" Dini norm of $\partial_{x_i x_j}^2 u$ and of Yu in terms of analogous norms of Lu and of u. The results discussed in this talk are obtained in collaboration with M. Bramanti (Politecnico di Milano) and B. Stroffolini (Università degli Studi di Napoli).

Alessia E. Kogoj (Università degli Studi di Urbino Carlo Bo)

"Liouville theorems for Kolmogorov-Fokker-Planck equations"

Several Liouville-type theorems will be presented, related to evolution equations on Lie groups and to their stationary counterpart. Our results apply in particular to the heat operator on Carnot groups, to linearized Kolmogorov operators and to operators of Fokker-Planck-type, like the Mumford operator. An application to the uniqueness for the Cauchy problem will be also shown. The results presented are contained in a series of papers in collaboration with A. Bonfiglioli, E. Lanconelli, Y.Pinchover, S. Polidoro and E. Priola.

Giampiero Palatucci (Università degli Studi di Parma)

"Boundedness estimates and Harnack inequalities for kinetic integral equations"

I shall present recent estimates obtained in collaboration with F. Anceschi and M. Piccinini for weak solutions to a class of kinetic equations with integro-differential diffusion in velocity.

Mirco Piccinini (Università degli Studi di Pisa)

"On the lack of compactness in the critical Sobolev embedding in the Heisenberg group"

In the sub-Riemannian setting of the Heisenberg group, we will investigate the effects of the lack of compactness in the critical Sobolev embedding. In particular, by means of variational techniques, we will show that optimal functions of a subcritical energy approximation of the Sobolev quotient in bounded (not necessarily regular) domains do concentrate energy at a single point. Moreover, assuming extra geometric features on the involved domain, in line with the particular underlying geometric framework, we show that the concentration point can be localized by the Green's function, thus proving that a conjecture of Brezis and Peletier (Essays in honor of Ennio De Giorgi 1989) does hold in the Heisenberg group.

Sergio Polidoro (Università degli Studi di Modena e Reggio Emilia)

"Taylor formulas for Nonlocal Kinetic Equations"

We consider spaces of Hölder continuous functions suitable for regularity theory for non local kinetic operators acting on the phase space. We prove an intrinsic Taylor-like formula, which naturally extends analogous formulas known for purely differential operators on Lie groups. These results have been proved in a joint work in collaboration with Maria Manfredini and Stefano Pagliarani.

Giulio Tralli (Università degli Studi di Ferrara)

"Gaining symmetries in the Heisenberg group: the case of bounded domains"

In this talk we discuss a family of overdetermined boundary value problems in the context of the Heisenberg group. We focus on various similarities and differences with respect to the classical symmetry result for the torsion function studied by Serrin. We show uniqueness results for gauge balls under suitable partial symmetry assumptions for the class of competitor sets. The main technical tool is a new Bochner-type identity for functions with toric/cylindrical invariances. This is a joint work with V. Martino.